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TO THE THEORY OF CHARGED PARTICLE SCATTERING BY COSMIC MAGNETIC FIELDS OF SIMPLEST TYPES

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TO THE THEORY OF CHARGED PARTICLE SCATTERING BY COSMIC MAGNETIC FIELDS OF SIMPLEST TYPES

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SUMMARY

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The propagation of cosmic rays in interstellar and interplanetary media is often viewed as a process of relativistic particle diffusion on magnetic inhomogeneities [1]. Considered in the present work are the scattering properties of magnetic fields of simplest configurations $H = \text{const}$ and $H \sim r^{-n}$, that would be adaptable to the diffusive process of charged particle propagation. Methods are developed for these scatterers, usually applicable in the theory of atomic collisions, that is, finding of the effective differential cross section, and also of total cross section.

A two-dimensional case is considered, whereby charged particles flow in a plane, and the vector of H is always directed perpendicularly to that plane.

Author

* * *

Effective Differential Cross Section of Charged Particle Scattering by a Uniform Magnetic Field. - It is shown in the collision theory that a total statistical characteristic can be given the process of collision with the help of the effective differential cross section $d\sigma$ (see for example [2]). For the case of two-dimensional scattering $d\sigma$ has the form

$$d\sigma = (dp / d\theta)\, d\theta, \tag{1}$$

where p is the impact parameter; θ is the angle of scattering.

* K TEORII RASSEYANIYA ZARYAZHENNYKH CHASTITS KOSMICHESKIMI MAGNITNYMI POLYAMI PROSTEYSHIKH TIPOV

We shall compute (1) for a particular case of scattering — the deflection of charged particles by a uniform magnetic field. The geometry of the collision is shown in Fig. 1. Here R_0 is the radius of a circle, inside which the magnetic field H = const (beyond this circle there is no magnetic field) ; β is the particle's angle of incidence; ρ is the curvature radius of the particle in the magnetic field **H**.

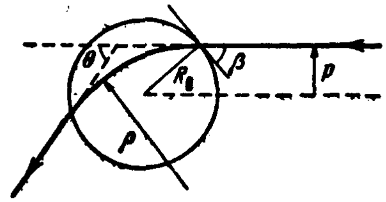

Fig. 1

$$\rho = mvc / ZeH, \tag{2}$$

where m, v, Ze are respectively the mass, the velocity and the charge of the particle; c is the speed of light. From geometrical considerations we have

$$\theta = 2\mathrm{arctg}\left(\frac{\sqrt{1-(p/R_0)^2}}{\alpha - p/R_0}\right) \qquad \left(\alpha = \frac{\rho}{R_0}\right). \tag{3}$$

Computing $dp / d\theta$ from (3) and substituting into (1), we find

$$d\sigma = \frac{1}{2} R_0 \sin\theta \left[\alpha + \sqrt{A} - \frac{1}{2\sqrt{A}} (1-\alpha^2)(1+\mathrm{tg}^2\,\theta/2 \right] d\theta, \tag{4}$$

$$\left(A = 1 + (1-\alpha^2)\,\mathrm{tg}^2 \frac{\theta}{2} \right).$$

Since A is under the radical, the following must be fulfilled :

$$\theta_{max} = 2\,\mathrm{arc\,tg}(\alpha^2 - 1)^{-1/2}. \tag{5}$$

Hence, at $\alpha < 1$ any scattering angles are possible, and for $\alpha > 1$ the scattering angles θ are bounded by the quantity

$$1 + (1-\alpha^2)\,\mathrm{tg}^2\,\theta / 2 \geqslant 0. \tag{6}$$

The dependence of the angle of scattering on the impact parameter computed by the formula (3) for various α, is plotted in Fig. 2.

In the particular case when $\alpha \ll 1$, , that is, when the intensity of the magnetic field is high, while the particle energy is low, formula (4) gives

$$d\sigma = \frac{1}{2} R_0 \sin \frac{\theta}{2} d\theta, \tag{7}$$

which coincides with the effective differential cross section of particle scattering on an infinitely hard sphere of radius R_0. The examples of scattering for particles of various energies are shown in Fig. 3

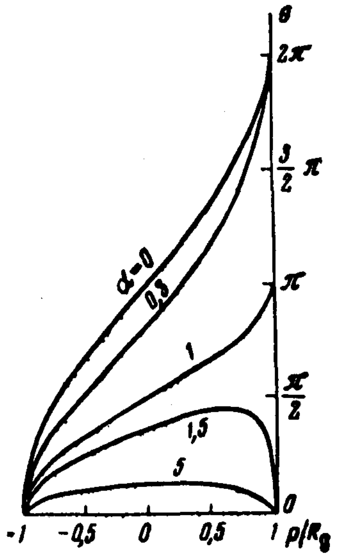

Fig. 2

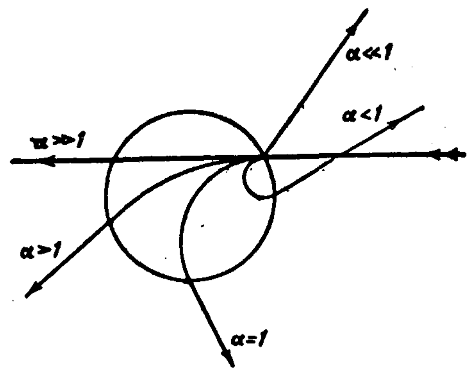

Fig. 3

<u>Diffusion Coefficient of Charged Particles at their Motion through Uniform Magnetic Fields</u>.- Let there be many uniform magnetic fields bounded by circles. These fields are distant from one another by an average value d. Particles diffuse through that system of magnetic fields by way of consecutive collisions. Let us find the diffusion coefficient $\varkappa$:

$$\varkappa = {}^1/_2 \lambda_{tr} v. \tag{8}$$

Here λ_{tr} is the transport mean free path; v is the velocity of the particle. Since the problem is two-dimensional, the multiplier is 2. The transport length of the free path may be written in the form [3]:

$$\lambda_{tr} = \frac{\lambda_s}{1 - \overline{\cos\theta}} = \frac{1}{N\sigma(1 - \overline{\cos\theta})} \qquad \left(N = \frac{1}{d^2}\right), \tag{9}$$

where N is the number of scatterers per unit of area; θ is the angle of scattering ($\cos\theta$ is averaged by the impact parameter); σ is the scattering cross section, which has the length scale for the two-dimensional case ; λ_s is the mean path of the particle between collisions; $\lambda_{tr} = \lambda_s$ only if there is isotropy of scattering at collision. If the scattering is anisotropic, that is if $\cos\theta \neq 0$, $\lambda_{tr} \neq \lambda_s$ too.

Let us find cos θ.

The unitary scattering of the particle is shown in Fig. 1. Inasmuch as the impact parameter p and the incidence angle β are linked by the correlation $p = R_0 \cos\beta$, and upon proper and simple trigonometric transformations formula (3) is brought to the form

$$\cos\theta = 1 - 2\frac{\sin^2\beta}{\alpha^2 - 2\alpha\cos\beta + 1}. \qquad (10)$$

It remains to integrate (10) over the angle of incidence from 0 to π (which corresponds to impact parameter variation from R_0 to $-R_0$) so as to find the mean value of $\cos\theta$.

$$\overline{\cos\theta} = 1 - \frac{2}{\pi}\int_0^\pi \frac{\sin^2\beta\, d\beta}{\alpha^2 + 1 - 2\alpha\cos\beta}. \qquad (11)$$

Taking this integral we obtain [4]

$$\overline{\cos\theta} = 0 \quad \text{at} \quad 0 < \alpha \leqslant 1, \qquad (12)$$
$$\overline{\cos\theta} = 1 - \frac{1}{\alpha^2} \quad \text{at} \quad 1 < \alpha < \infty.$$

Substituting (12) into (11), we obtain

$$\lambda_{tr} = \lambda_s = \frac{d^2}{2R_0} \quad \text{at} \quad 0 < \alpha \leqslant 1 \qquad (13)$$

and, consequently, according to formula (8)

$$\varkappa = vd^2/4R_0, \qquad (14)$$

$$\lambda_{tr} = \alpha^2\lambda_s = \frac{d^2}{2R_0^3}\left(\frac{mvc}{zeH}\right)^2 \qquad 1 < \alpha < \infty. \qquad (15)$$

The diffusion coefficient is then equal to

$$\varkappa = \frac{vd^2}{4R_0^3}\left(\frac{mvc}{zeH}\right)^2. \qquad (16)$$

For ultrarelativistic particles $v \to c$ and $mvc \to E$, where E is the particle's total energy. Hence

$$\varkappa = \frac{cd^2}{4R_0^3}\left(\frac{E}{zeH}\right)^2. \qquad (17)$$

The magnetic field **H** vector, perpendicular to the propagation plane of charged particles, may have two directions, differing by the angle π.

In the given case with great α the diffusion will take place only when the magnetic fields with both possible directions H are encountered equally frequently. But whenever there are only magnetic fields of single direction, the charged particles, deflecting at each collision toward the same side, can no longer egress from the bounded region of space.

Diffusion Coefficient of Charged Particles at Motion through Magnetic Fields of the Form $H = M/r^n$. Let again magnetic formations be disposed at average distance d from one another. Now, however, the intensity of the magnetic field H decreases from the center according to the law $H = M/r^n$, where M is a constant factor and r is the distance from the magnetic formation.

We shall estimate that the distance, over which the field H affects substantially the motion of the charged particle, is much less than d. Let us find the diffusion coefficient.

Let us consider the motion of the charged particle in the magnetic field $H = M/r^n$. We obtain at once from the law of torque [5], the equation for the trajectory of the particle

$$r^2\frac{d\theta}{ds} + \frac{eM}{(2-n)r^{n-2}mvc} - \frac{ec_1}{mvc} = \frac{c_2}{mv} \qquad (n \neq 2), \qquad (18)$$

$$r^2\frac{d\theta}{ds} + \frac{eM}{mvc}\ln r - \frac{ec_1}{mvc} = \frac{c_2}{mv} \qquad (n = 2), \qquad (18a)$$

where r, θ are the cylindrical coordinates of particle motion; ds is an element of trajectory length; c_1 and c_2 are constants. Since $r^2\,(d\theta/ds)$ in (18) has the length dimensionality, any other addend in this equation has same. Hence $eM/r^{n-2}mvc$ has the dimensionality of length, and, consequently, the quantity $(eM/mvc)^{1/(n-1)}$ has the same dimensionality. If we introduce a new unit of length*

$$l = (eM/mvc)^{1/(n-1)}, \qquad (19)$$

* In the particular case when $n = 3$, is the Stormer unit (see below).

the equations (18) will be dimensionless, that is, (18) will be written in the form

$$r_1^2 \frac{d\theta}{dS_1} + \frac{1}{(2-n)\, r_1^{n-2}} = \text{const}, \tag{20}$$

where $r_1 = r/l$ and $S_1 = S/l$.

Let us apply the unit of length l, introduced during the consideration of particle trajectory in the field $H = M/r^n$, for the description of the particle scattering process by such a field.

We shall express the scattering cross section in units of l. Then from formulas (8) and (9), we shall find the diffusion coefficient

$$\varkappa = \frac{a^2 v}{2\sigma\overline{(1-\cos\theta)}} \sqrt[n-1]{\frac{mvc}{eM}}.$$

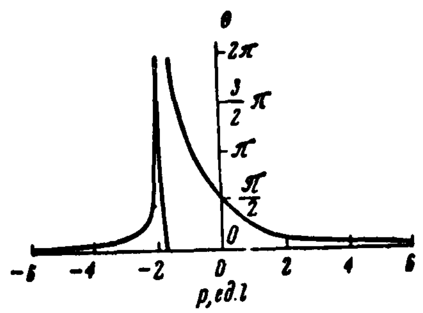

Fig. 4

The constant $\sigma\overline{(1-\cos\theta)}$ may be found if the trajectories of motion for various "impact parameters" are known. It should be pointed out that the equations of particle motion in fields of the form $H = M/r^n$ are not integrable in simplest functions and require for their solution approximate methods.

At $n = 1$, the quantity l as the unit of length loses its sense. This implies that the fields of type $H = M/r^n$ are valid for charged particle scatterings at any value of n.

As may be seen, the particle can not egress from the field $H = M/r^n$ at $n < 1$. In this case the centrifugal force $mv^2/r \sim 1/r$ is decreasing with distance faster than H. The retention and the focusing of electrons by the fields $H = M/r^n$ with $n < 1$ is utilized in betatrons. (See [6]).

If $n > 1$, the centrifugal force decreases with distance slower than H and the particle may drift to infinity. Consequently the scattering and diffusion processes of charged particles by fields $H = M/r^n$ can take place only for $n > 1$.

At $n = 3$, the magnetic field $H = M/r^3$, which corresponds to the distribution of the field in the equatorial plane of the magnetic dipole.

The unit of length l takes the form

$$l = \sqrt{eM/mvc}. \tag{21}$$

Now M has the sense of a magnetic dipole. If M is equal to the Earth's magnetic moment, this unit is sometimes called "Stormer". The author himself constructed all trajectories precisely in these units of length [7] (see also [8]).

Utilizing the trajectories of charged particle motion in the equatorial plane, of the magnetic dipole, obtained by numerical integration in [9], we constructed the dependence of the scattering angle θ on the impact parameter p (Fig. 4). For great values of the parameter p we have $\theta = \theta_0 (p_0/p)^2$, where p_0 is the value of the parameter at small deflection of θ_0.

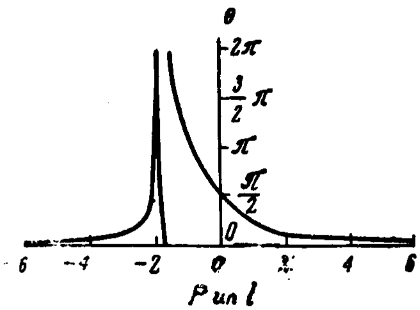

Fig. 4

Inasmuch as for $p \gg 1$, we have $dp/d\theta = \frac{1}{2} p_0 \theta^{1/2} \theta^{3/2}$, then, according to (1), $d\sigma = \frac{1}{2} p_0 \theta_0^{1/2} \theta^{-3/2} d\theta$ and the averaging in (20) gives for $p > p_0$ (taking into account that $\theta_0 \ll 1$)

$$\overline{\sigma(1-\cos\theta)}_{p>p_0} = p_0\sqrt{\theta_0}\int_0 \theta^{-3/2}(1-\cos\theta)\,d\theta = p_0\frac{\theta_0^2}{3}. \tag{22}$$

The averaging for $p < p_0$ is effected numerically according to Fig. 4. In the final result we have

$$\varkappa = (0{,}18 \pm 0{,}02)\, d^2 v \sqrt{mvc/eM}. \tag{23}$$

For ultrarelativistic energies

$$\varkappa = (0{,}18 \pm 0{,}02)\, d^2 c \sqrt{E/eM}. \tag{24}$$

**** THE END ****

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	NESS	
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	WHITE	
615	BOURDEAU	
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